Generalized atmospheric dispersion correctors for the Thirty Meter Telescope

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ABSTRACT

The Thirty Meter Telescope (TMT) is unbaffled and has stability requirements tighter than the previous generation of 10-m class telescopes, leading to tougher requirements on atmospheric dispersion correctors (ADCs). Since instruments are internally baffled, ADCs may no longer shift the position of the telescope exit pupil. Designs that control pupil position are explored.

Keywords: atmospheric dispersion correctors, astronomical telescopes, telescopes

1 INTRODUCTION

When light enters the atmosphere at an angle, the air disperses the color slightly in angle. This is an issue for spectrometers, since the light will not all go into a narrow slit. An atmospheric dispersion corrector (ADC) is used to correct this. Usually an ADC consists of several wedges of glass, sometimes of different types of glass, which translate or rotate to actively correct the atmospheric dispersion, as a function of the elevation angle of the telescope.

Another issue is anamorphic distortion, which is important for a wide-field instrument. The atmospheric refraction compresses the field in the zenith direction. For imaging, this effect can be corrected in software, but for spectroscopy with a wide-field slit mask, the objects move relative to the slits as the field rotates. The ADC can also be used to correct this effect.

On the telescope side, TMT is being built without baffles in order to decrease wind loading, and all instruments are expected to internally baffle for stray light. This means that an instrument must first have a cleanly formed internal pupil, and the pupil of the telescope must be stationary relative to the internal baffling. The second issue is that the TMT tertiary mirror can tip and tilt, but not actively translate. Gravitational deformation of the structure leads to translation of the tertiary, which translates both the pupil and the image. This paper addresses both, since some ADC designs do not maintain a stationary pupil.

2 ATMOSPHERIC REFRACTION AND DISPERSION

To first order, the atmosphere is a big plate of air above the telescope. The light entering the atmosphere at an angle is refracted and dispersed just like a prism (Figure 1). For a single-slit spectrometer, the dispersion has the effect that the light is spread out as a function of wavelength and may not all pass through the slit, depending on the angle of the slit to the horizon. A second, more obscure effect is that the atmospheric refraction causes anamorphic distortion of the sky image, so the plate scale is different depending on the angle to the horizon. For long observations using wide-field imaging spectrometers, the effect can also cause light to miss the slit as a function of the horizon angle.

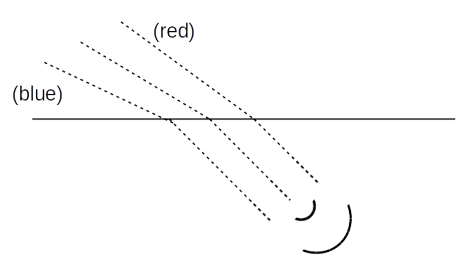

Figure 1: atmospheric dispersion bends short wavelengths more

For typical temperature, pressure, and humidity of TMT located at Mauna Kea, the deviation of a sky object at 65° zenith angle is 77 arcseconds at a wavelength of 0.5 microns. The dispersion is 7.3 arcseconds/micron, and the anamorphic distortion is 1.001[1,3]. For the 20-arcminute field of view of TMT, this anamorphic distortion leads to a relative image motion of 1.2 arcseconds, which is significant when compared to the typical 0.75-arcsecond slit used for spectrometry.

3 TELESCOPE ABERRATIONS

TMT is a classical Richey-Crétien Cassegrain telescope with a flat tertiary mirror that repositions the telescope focal surface to instruments on the Nasmyth platform. Since the telescope has no baffling to keep stray light from the focal surface, instruments are expected to internally baffle for stray light by having an internal stop conjugate to the telescope exit pupil. If the tertiary mirror rotates about the intersection point between the telescope axis and the elevation bearing, then any motion of the tertiary repositions the focus and the telescope exit pupil together.

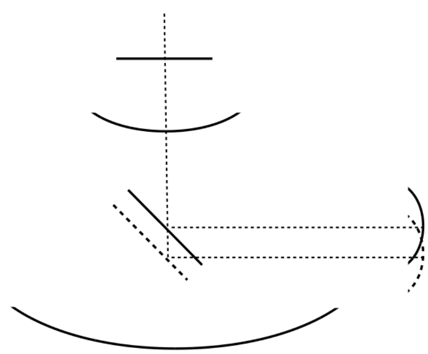

Figure 2: Tertiary translation moves the image, the pupil, and the field curvature in the azimuth direction

This happy story breaks down in practice because TMT is a very large steel structure which flexes under gravity differently for each telescope elevation angle, so the tertiary does not remain centered on the intersection of the two previously mentioned axes. If the tertiary is translated orthogonal to its own surface (in piston), then the telescope focus and the apparent exit pupil position both move at the instrument focal surface (Figure 2). For an instrument located on the elevation axis, this motion is in azimuth. For other instrument locations, this apparent motion at some angle which depends on the instrument location on the Nasmyth platform.

In practice, the telescope control system will automatically react to keep the guide object on the guider. If the tertiary is rotated to keep the pupil on axis, then result is that the telescope is operating off-axis. Since a Cassegrain telescope has field curvature, the difference between the before and after focal surfaces is a defocus that is linear across the entire field of view. For a wide-angle instrument, this is not acceptable. One possible way to correct this is to rotate the secondary about the coma-free point; however, this introduces binodal astigmatism, which is also an aberration which goes linearly with field angle for small field angles. Even if the binodal astigmatism is small, it will be a function of elevation angle and therefore change throughout an observation, making calibration difficult. The telescope aberrations are all constant if the focus shift is corrected by the ADC, rather than by repointing the telescope.

If the tertiary is moved perpendicular to it surface by a distance E, then the focus moves sideways in the azimuth direction by a distance $E/\sqrt{2}$. The pupil angle is shifted by $E/(\sqrt{2}L)$, where L (=46.4m for TMT) is the distance from the focus to the pupil. If the focal length is f (=450m for TMT), then the telescope re-points by an angle $E/(\sqrt{2}f)$. Rotating the tertiary by an angle τ moves the pupil by 2τ and the focus by $2\tau B$, where B (=20m for TMT) is the distance from the focus to the tertiary. If the curvature of the focal surface is C (=1/(3m) for TMT), then the linear defocus across the field is $\frac{1}{2} Cx^2 - \frac{1}{2} C(x-E/\sqrt{2})^2 = CEx/\sqrt{2}$ plus a constant offset, where x is the image motion in meters.

4 TYPICAL ADC'S

ADCs are, by and large, made from wedges of glass that act as prisms. These wedges can be either single materials or glued-up stacks of materials with differing refractive properties. Figure 3 below shows one popular type of ADC, commonly called a Risley prism. The two wedges are close to each other and counter rotate. When the wedges are opposite, as in the left side of Figure 3, any dispersion created by the first surface is canceled by the last surface. When the wedges are rotated about the optical axis so that the thin edges are together, then dispersion (and deviation) is maximized.

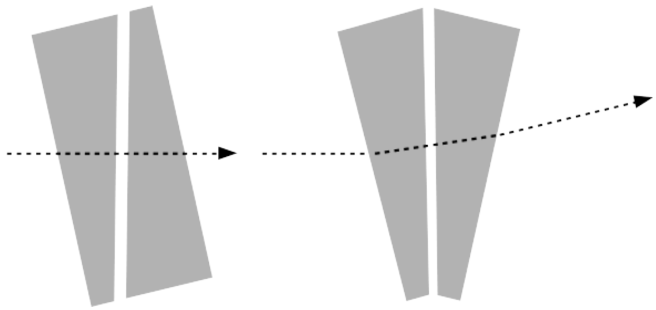

Figure 3: Counter-rotating ADC (Risley prism)

The other common ADC design is the "trombone" ADC, where one wedge is fixed and the other wedge translates along the space between the fixed wedge and the focal surface. This is shown in Figure Figure 4 below. When the moving wedge is touching the fixed wedge, the system has no dispersion or deviation. When the moving wedge is at the focal surface, it has virtually no dispersive effect on the images, and so the maximum dispersion (due to the fixed wedge) occurs.

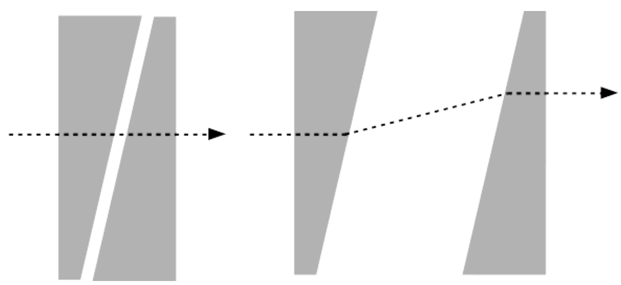

Figure 4: Trombone ADC

Both of the above designs have the desired variable dispersion, while only having one degree of freedom. Alternate designs may be able to compensate for more atmospheric and telescope mechanical effects.

5 OPTICAL CONSIDERATIONS

Taking a perfectly good telescope and sticking a few wedges of glass into the beam has all sorts of side effects besides canceling the atmospheric dispersion. The effects considered here are:

- The telescope image has anamorphic distortion (canceling the atmosphere).
- The telescope image is dispersed (canceling the atmosphere).
- The telescope image is displaced (also caused by telescope guiding).
- The telescope exit pupil is dispersed.
- The telescope exit pupil is displaced (also caused by tertiary guiding).
- Linear defocus across the field (also caused by telescope pointing).

The effects not considered here are:

- 3rd order aberrations caused by adding optical material to the light path
- Second-order effects (ie, assume the effects of individual wedges linearly add)
- Stray light, surface quality, index inhomogeneity, scatter, absorption

6 BACKGROUND MATHEMATICS

We begin my assuming a stack of wedges of glass, and that all of our angles are small enough that we can linearize Snell's Law. Since each astronomical observation has some unique wavelength passband, we correct the atmospheric dispersion at two wavelengths, λ_1 and λ_2. Define three functions of the index of refraction of the wedge material $n(\lambda)$ as

$$n=\frac{n(\lambda_1)+n(\lambda_2)}{2}, \quad (1$$

$$\Delta n=n(\lambda_1)+n(\lambda_2)-2, \quad (2)$$

and

$$\delta n=\frac{n(\lambda_1)-n(\lambda_2)}{\lambda_2-\lambda_1}. \quad (3)$$

The beam deviation P for the i^{th} wedge is then

$$P_i=\Delta n_i A_i. \quad (4)$$

where A_i is the wedge angle. Each wedge has two surfaces, but the factor of two has been absorbed into the construction of Δn. Here both P_i and A_i are 2-dimensional vectors in a plane parallel to the focal surface, so as a function of rotation angle θ about the instrument rotation z-axis, A_i is

$$A_i=|A_i|\begin{bmatrix}\cos(\theta_i)\\ \sin(\theta_i)\end{bmatrix} \text{where the coordinates are} \begin{bmatrix}\text{azimuth}\\ \text{zenith angle}\end{bmatrix}. \quad (5)$$

The wedge also has dispersion, which has a similar formula,

$$p_i=\delta n_i A_i. \quad (6)$$

The location of the telescope pupil, as seen from the instrument, only depends on the beam deviation. If the pupil location is specified as the angular motion of the center of the pupil relative to the z-axis, as seen by the focal surface, then the pupil motion is the same as the beam deviation, and the pupil dispersion is the same as the beam dispersion.

The location of the focus at the instrument is shifted by each wedge in proportion to the distance from the wedge to the focal surface z_i, so

$$F_i=\frac{1}{f} z_i \Delta n_i A_i \quad (7)$$

and similarly for the dispersion,

$$d_i=\frac{1}{f} z_i \delta n_i A_i. \quad (8)$$

The f here is the focal length of the telescope, which has been added so that F_i and d_i have units of angle on the sky.

The linear defocus from a wedge is the change in optical path length through the wedge,

$$H_i = ½\,\Delta n_i A_i \tag{9}$$

where the defocus at distance x from the center would be $H_i x$.

The anamorphic distortion of a wedge depends on the tilt of the wedge about the axis of the wedge's dihedral angle. If the wedge tilt is symmetric (as a minimum-deviation prism), then there is no anamorphic distortion. Call this tilt angle φ=0. On the other hand, if the wedge is rotated so one face is normal to the z-axis, then $\varphi = ½|A|$, while the anamorphic distortion is $\sec(A+P) \approx 1 + ½\,(nA)^2$. So the anamorphic distortion from a wedge is

$$S_i - 1 = n_i^2 \phi_i A_i\,. \tag{10}$$

The anamorphic distortion from two wedges multiplies, but once second-order terms are discarded, this is equivalent to adding up the right-hand sides of equation (10).

For an ADC made up of a set of wedges, the total ADC effect is then

$$\begin{aligned}
P &= \sum_i \Delta n_i A_i && \text{(pupil shift)}\\
p &= \sum_i \delta n_i A_i && \text{(pupil dispersion)}\\
F &= \frac{1}{f}\sum_i z_i \Delta n_i A_i && \text{(pointing shift)}\\
d &= \frac{1}{f}\sum_i z_i \delta n_i A_i && \text{(dispersion)}\\
H &= ½\sum_i \Delta n_i A_i && \text{(linear defocus)}\\
S &= 1 + \sum_i n_i^2 \phi_i A_i && \text{(scale change).}
\end{aligned} \tag{11}$$

From the previous sections, the left-hand sides of these equations are (for TMT) desired to be

$$\begin{aligned}
P &= \begin{bmatrix} \sqrt{2}\,E/L + 2\tau_x + \alpha_x \\ 2\tau_y + \alpha_y \end{bmatrix}\\
p &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}\\
F &= \begin{bmatrix} \sqrt{2}\,E/f + 2\tau_x B/f + \alpha_x \\ 2\tau_y B/f + \alpha_y \end{bmatrix}\\
d &= \begin{bmatrix} 0 \\ d_{sky} \end{bmatrix}\\
H &= \begin{bmatrix} 2Cf\,\alpha_x \\ 2Cf\,\alpha_y \end{bmatrix}\\
S &= \begin{bmatrix} 1 \\ S_{sky} \end{bmatrix}.
\end{aligned} \tag{12}$$

Other telescopes such as GMTO or E-ELT would have different requirements. Here τ_x, τ_y are the azimuth and zenith-angle tilts of the tertiary, while α_x, α_y are the azimuth and zenith-angle motions of the telescope on the sky. Equations (11) and (12) combine (completely ignoring signs) to give

$$
\begin{aligned}
\sqrt{2}E/L+2\tau_x+\alpha_x &= \sum_i |A_i|\cos(\theta_i)\Delta n_i & 2\tau_y+\alpha_y &= \sum_i |A_i|\sin(\theta_i)\Delta n_i \\
0 &= \sum_i |A_i|\cos(\theta_i)\delta n_i & 0 &= \sum_i |A_i|\sin(\theta_i)\delta n_i \\
\sqrt{2}E+2\tau_x B+f\alpha_x &= \sum_i |A_i|\cos(\theta_i) z_i \Delta n_i & 2\tau_y B+f\alpha_y &= \sum_i |A_i|\sin(\theta_i) z_i \Delta n_i \\
0 &= \sum_i |A_i|\cos(\theta_i) z_i \delta n_i & f d_{atm} &= \sum_i |A_i|\sin(\theta_i) z_i \delta n_i \\
4Cf\alpha_x &= \sum_i |A_i|\cos(\theta_i)\Delta n_i & 4Cf\alpha_y &= \sum_i |A_i|\sin(\theta_i)\Delta n_i \\
0 &= \sum_i |A_i|\cos(\theta_i) n_i^2 \phi_i & S_{atm}-1 &= \sum_i |A_i|\sin(\theta_i) n_i^2 \phi_i .
\end{aligned}
\tag{13}
$$

7 SINGLE-MATERIAL CORRECTORS

The TMT focal surface is about 2.6 meters in diameter. The only refractive material that can reasonably be sourced in this size is fused silica. Under the assumption that all the wedges are made from the same material, equation (11) simplifies to

$$
\begin{aligned}
P/\Delta n &= \sum_i A_i \\
p/\delta n &= \sum_i A_i \\
Ff/\Delta n &= \sum_i z_i A_i \\
df/\delta n &= \sum_i z_i A_i \\
2H/\Delta n &= \sum_i A_i \\
(S-1)/n^2 &= \sum_i A_i \phi_i .
\end{aligned}
\tag{14}
$$

Note that *P*, *p*, and *H* have become degenerate, as well as *F* and *d*. The conclusion is that an ADC made from a single material cannot control the deviation and the dispersion separately, either for the pupil or for the image. This is regardless of the number of wedges in the ADC. The only algorithm available is then:

1) Set ADC angles and z-axis positions to correct atmospheric dispersion (*d*) with no pupil dispersion (*p*)
2) Set telescope and tertiary to correct image offset (*F*) and pupil offset (*P*)
3) Set ADC wedge tilts to correct anamorphic distortion (*S*)

This is essentially the classical ADC control algorithm, where the ADC corrects zenith-angle effects, while the telescope corrects azimuth effects. The ADC is not able to offset linear defocus from the telescope.

8 TWO-WEDGE CORRECTORS

Assuming only two wedges, equation (13) simplifies to give

$$
\begin{aligned}
\sqrt{2}E/L+2\tau_x+\alpha_x &= \Delta n_1|A_1|\cos(\theta_1)+\Delta n_2|A_2|\cos(\theta_2) & 2\tau_y+\alpha_y &= \Delta n_1|A_1|\sin(\theta_1)+\Delta n_2|A_2|\sin(\theta_2) \\
0 &= \delta n_1|A_1|\cos(\theta_1)+\delta n_2|A_2|\cos(\theta_2) & 0 &= \delta n_1|A_1|\sin(\theta_1)+\delta n_2|A_2|\sin(\theta_2) \\
\sqrt{2}E+2\tau_x B+\alpha_x &= z_1\Delta n_1|A_1|\cos(\theta_1)+z_2\Delta n_2|A_2|\cos(\theta_2) & 2\tau_y B+\alpha_y &= z_1\Delta n_1|A_1|\sin(\theta_1)+z_2\Delta n_2|A_2|\sin(\theta_2) \\
0 &= z_1\delta n_1|A_1|\cos(\theta_1)+z_2\delta n_2|A_2|\cos(\theta_2) & f d_{atm} &= z_1\delta n_1|A_1|\sin(\theta_1)+z_2\delta n_2|A_2|\sin(\theta_2) \\
4Cf\alpha_x &= \Delta n_1|A_1|\cos(\theta_1)+\Delta n_2|A_2|\cos(\theta_2) & 4Cf\alpha_y &= \Delta n_1|A_1|\sin(\theta_1)+\Delta n_2|A_2|\sin(\theta_2) \\
0 &= n_1^2|A_1|\cos(\theta_1)\phi_i+n_2^2|A_2|\cos(\theta_2)\phi_2 & S_{atm}-1 &= n_1^2|A_1|\sin(\theta_1)\phi_1+n_2^2|A_2|\sin(\theta_2)\phi_2 .
\end{aligned}
\tag{15}
$$

The two requirements for the pupil dispersion on the second line result in two conditions,

$$
\begin{aligned}
\delta n_1|A_1| &= \delta n_2|A_2| \\
\theta_1 &= \theta_2+180° .
\end{aligned}
\tag{16}
$$

The conclusions from this are

- Requiring no pupil dispersion requires the trombone design; counter-rotating wedges disperse the pupil.
- Because δn depends on wavelength, the two wedges must be identical if the ADC is to work perfectly at arbitrary wavelength bandpasses.

Using (16) and assuming $\theta_1 = 90°$ to simplify (15) gives

$$\begin{array}{lll} \sqrt{2}E/L+2\tau_x=0 & 2\tau_y+\alpha_y=\Delta n_1|A_1|-\Delta n_2|A_2| & (P) \\ \sqrt{2}E+2\tau_x B=0 & 2\tau_y B+\alpha_y=z_1\Delta n_1|A_1|-z_2\Delta n_2|A_2| & (F) \\ & f\,d_{atm}=\delta n_1|A_1|(z_1-z_2) & (D) \\ \alpha_x=0 & 4Cf\,\alpha_y=\Delta n_1|A_1|-\Delta n_2|A_2| & (H) \\ & S_{atm}-1=n_1^2|A_1|\phi_1-n_2^2|A_2|\phi_2 & (S) \end{array} \qquad (17)$$

The two-wedge ADC has no effect on the azimuth direction at all. The control algorithm is then

1) set the distance between the two wedges in linear proportion to the image dispersion (*d*)
2) set telescope α_y to cancel the ADC linear defocus (*H*).
3) set the tertiary τ_y to cancel the telescope and ADC pupil shift (P)
4) slide the entire ADC in z to cancel the focus shift (*F*)
5) set the wedge tilts to correct the anamorphic distortion (*S*).

So a two-wedge, multi-material ADC can, in theory, solve all the problems in the zenith direction, the performance is not really significantly better than a single-material ADC, if at all.

9 MULTI-WEDGE CORRECTORS

The process of using sines and cosines as in the two-wedge design clearly becomes a morass for multi-wedge ADCs. Instead, a graphical approach makes more sense. Consider each term in the summations as a vector in (x,y) or (azimuth,elevation) space. The length of a vector term for, say, *p*, is $\delta n|A|$, and the direction is θ. These vectors can be placed end-to-end on a 2-D (x,y) plot to represent the summations. Since *p* is a zero-length vector, the vectors created a closed, directed polygon. Thus for a two-wedge ADC, the vectors must be equal and opposite, exactly as in equation (16). See the left-hand side of Figure 5 for what these graphs look like. The short arrow in the *d*-plots represents the atmospheric image dispersion to be matched by the ADC.

For the three-wedge ADC, the three vectors for *p* form a triangle. Given that the lengths of the vectors are fixed by the materials and shapes of the elements, the only modification that can be made to the wedge positions while keeping *p* zero is to rotate the triangle as a whole. Since this rotation is reserved for aligning the dispersion with the elevation direction, the wedges cannot independently rotate.

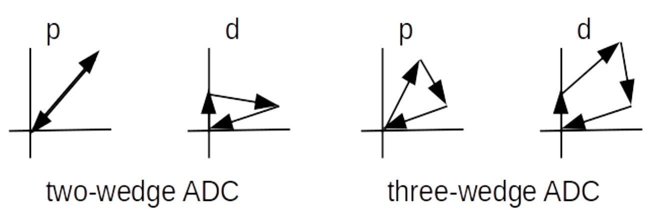

Figure 5: graphical illustration of ADC pupil and image dispersion

The conclusion is that a three-wedge ADC is only marginally better than a two-wedge ADC, in that the extra z-motion of the extra wedge allows for correction of focus shift without moving the entire ADC. Also, the pupil dispersion can most likely be corrected over an arbitrary passband. By considering directed graphs, it is clear that a 4-wedge ADC has significantly more flexibility compensate for various issues, but for a wide field of view, a four-wedge, multi-material ADC is quite unrealistic.

10 CONCLUSIONS

This paper considers generalized ADCs on the TMT to see if the ADC can compensate not only for atmospheric dispersion, but also for the translation of the tertiary. The conclusion is that no ADC with less than 4 wedges will compensate for the tertiary, since the tertiary motion is orthogonal to the dispersion direction. A trombone ADC made from a single material (or identical combinations of materials) can compensate the dispersion over any passband without dispersing the pupil. Using multiple materials or an extra wedge does not significantly improve the ADC performance.

11 REFERENCES

[1] Stone, Ronald C. “An accurate Method for Computing Atmospheric Refraction”, PASP, **108**, 1051-1058 (1996).
[2] Filippenko, A. V., “The importance of atmospheric differential refraction in spectrophotometry”, PASP, **94**, 715-721 (1982).
[3] Sutin, B. M., Skewray, <https://github.com/skewray/skewray/blob/master/test/atmosphere.sky>, (2016).

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